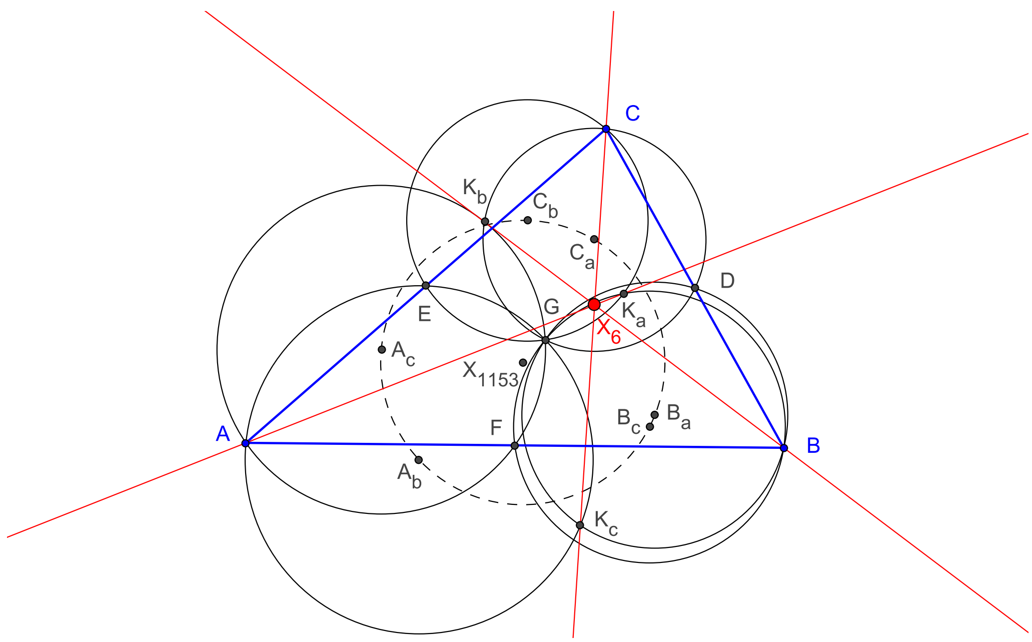
Floor's Monthly problem

A triangle is divided by its three medians into 6 smaller triangles. The circumcenters of these smaller triangles are concyclic. Their circle, the Van Lamoen circle, is introduced in Floor van Lamoen Problem 10830, American Mathematical Monthly 107 (2000) 863; solution by the editors, 109 (2002) 396-397.

Let ABC be a triangle and G its centroid. The midpoints of sides BC, CA, AB are labeled D, E, F. The circumcenters A_b, A_c, B_c, B_a, C_a, C_b of the 6 triangles AGE, AGF, BGF, BGD, CGD, CGE lie on the Van Lamoen Circle. Its center is X_{1153}.

Points of intersection of the circumcircles of these triangles, other than G:
$K_a = (B_c) \cap (C_b), K_b = (C_a) \cap (A_c), K_c = (A_b) \cap (B_a)$;
$G_{ab} = (A_b) \cap (C_a)$, the symmetrical point de G w/r to the line A_bC_a,
$G_{ac} = (A_c) \cap (B_a)$, the symmetrical point de G w/r to the line A_cB_a,
$G_{bc} = (B_c) \cap (A_b)$,
$G_{ba} = (B_a) \cap (C_b)$,
$G_{ca} = (C_a) \cap (B_c)$,
$G_{cb} = (C_b) \cap (A_c)$.

1. The lines AK_a, BK_b, CK_c concur in the symmedian, X_6.



2. If A', B' and C' are the points of intersection of the lines (A_bC_a, A_cB_a), (B_cA_b, B_aC_b) and (C_aB_c, C_bA_c), respectively, then AA', BB', CC' concur in the triangle center with (6-9-13)-search number 1.88868384192281470263 and barycentric coordinate:

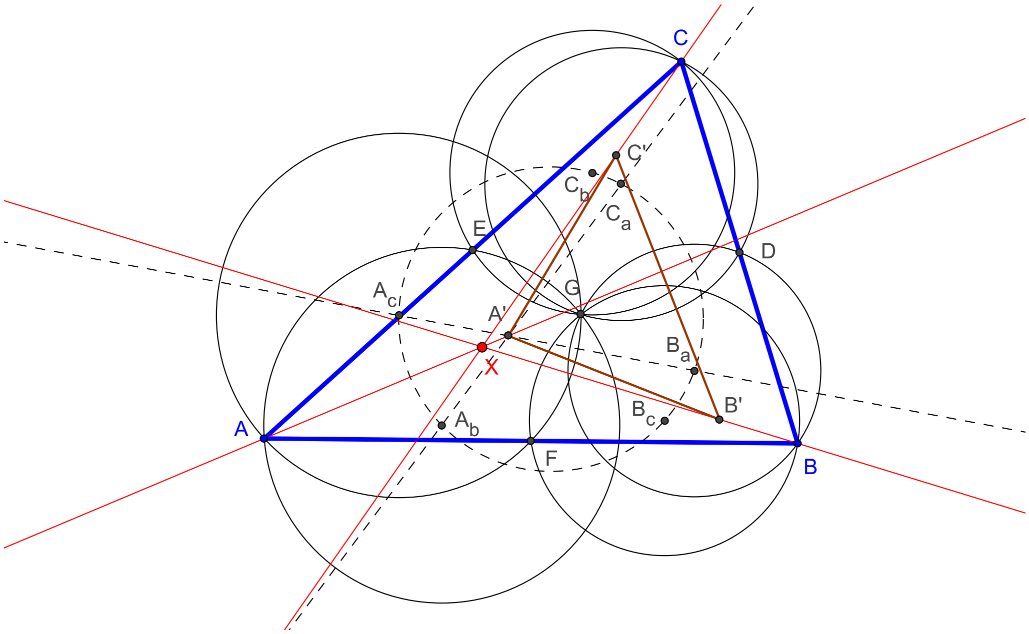
$$\left(\frac{1}{S_A^2 - S_BS_C - 2S^2} : \frac{1}{S_B^2 - S_CS_A - 2S^2} : \frac{1}{S_C^2 - S_AS_B - 2S^2}\right).$$

Is the isogonal conjugate of the X_{576}, orthoharmonic of X_{187}.

Note: The triangle center of the first barycentric coordinate

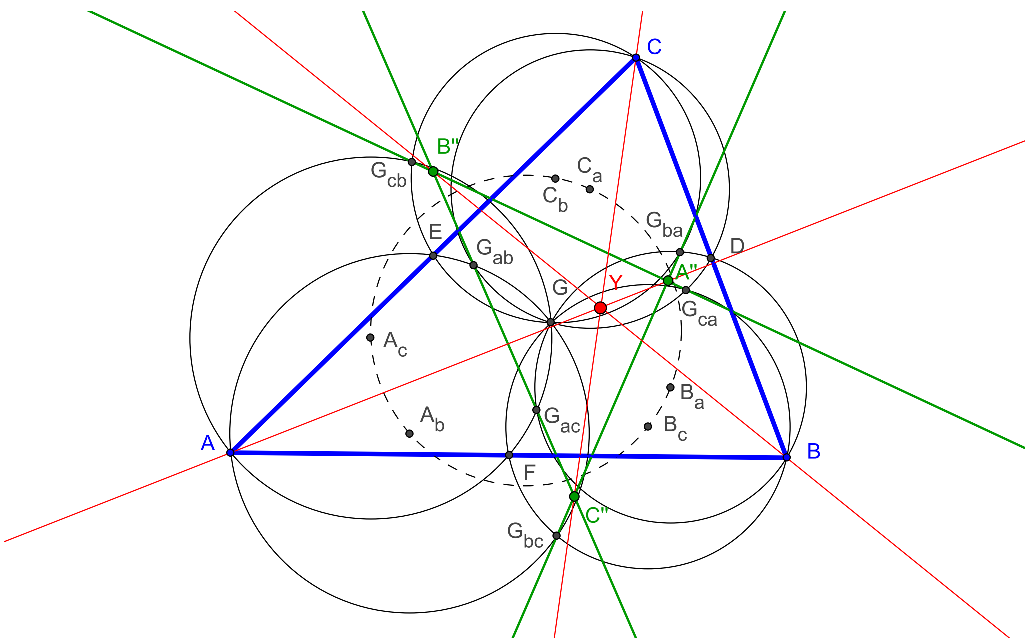
$$\frac{1}{S_A^2 - S_BS_C + 2S^2}$$

is X_{262} = isogonal conjugate of midpoint of Brocard diameter (X_{182}).



3. The lines $G_{ab}G_{ac}, G_{bc}G_{ba}, G_{ca}G_{cb}$ determine a triangle $A''B''C''$ perspective with ABC, and the coordinates of the perspector are:

$$\left(\frac{1}{(2a^2-b^2-c^2)^2-9b^2c^2} : \frac{1}{(2b^2-c^2-a^2)^2-9c^2a^2} : \frac{1}{(2c^2-a^2-b^2)^2-9a^2b^2}\right).$$



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